# The reverse DTML-conversion model with its mathematical formulas 

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ISSN: 2243-772X
Received: 1 April 2013
Revised: 4 January 2014
DOI: 10.5861/ijrsc.2014.419

Accepted: 14 February 2014 Online ISSN: 2243-7797 OPEN ACCESS

## Abstract

Conversion within quantities of same units and between quantities of different units is a thorny subject to students of Jasikan College of Education (Butterfield, Sutherland, \& Molyneux-Hodgson, 2000) and its treatment by tutors sometimes becomes very difficult such that most tutors resort to handling the subject theoretically/ abstractly. When this happens most students seemed not to comprehend the subject. In view of this, the Reverse DTML-Conversion model (i.e. The Reverse D-Conversion model, The Reverse T-Conversion model, The Reverse M-Conversion model and The Reverse L-Conversion model) was designed. The Reverse DTML-Conversion Model is a model that has been designed by the researcher to make the teaching of conversion in measurement very easy to tutors and meaningful to students.

Keywords: The Reverse D-Conversion model; The Reverse T-Conversion model; The Reverse M-Conversion model; The Reverse L-Conversion model; mathematical formula

## The reverse DTML-conversion model with its mathematical formulas

## 1. Introduction

The Reverse DTML-Conversion Model is a model that has been designed by the researcher to make the teaching of conversion in measurement very easy to tutors and meaningful to students. This employs the use of a straight line divided into equal sections/parts depending on the physical quantity of concern. The three fundamental/basic physical quantities which are used in this model are distance, time and mass. Conversion within quantities of same units and between quantities of different units is a thorny subject to students (Butterfield, Sutherland \& Molyneux-Hodgson, 2000) and its treatment by tutors sometimes becomes very difficult such that tutors resort to handling the subject theoretically/abstractly. When this happens most students seemed not to comprehend the subject.

In view of this, Trimpe studies in 2000 and 2008 and http://www.bayhicoach.com designed a metric mania and metric conversion: stair-step method to enable students understands conversion of distance, litre (volume) and grams (mass). Trimpe metric mania and the stair-step method even though very similar were good; however, it concentrated on only one dimension distance (i.e. considering distance) and just one aspect of the three dimension distance i.e. litre, while the Reverse DTML-Conversion extends from one distance dimension to two distance dimension and three distance dimension (i.e. considering distance) and also each step in the metric mania is in multiples of decimals (i.e. 0.1) when dealing with smaller units of the basic unit and in multiples of ten (i.e. 10) when dealing with higher units also of the basic unit (Appendix A and Appendix B) and also from http://www.bayhicoach.com that steps in metric system are all in multiples of ten (10) and decimal (0.1).

However, with the Reverse DTML-Conversion model, all steps with respect to length (i.e. one dimension distance), mass and litre S.I units are in multiples of ten (10). All steps with respect to Area (i.e. two dimension distance) are in multiples of hundreds (100), all steps with respect to volume (i.e. three dimension distance) are in multiples of thousands (i.e. 1000), and all steps with respect to time are in multiples of sixty (60). All units on the number line started with zero mark through ten steps, hundred steps, thousand step and sixty steps each, and also in place of the basic unit for both higher units and smaller as in metric mania and the stair-step have been replaced with the smallest unit and the highest unit on a straight line in ten steps, hundred steps, thousand step and sixty steps.

The Reverse DTML-Conversion model was designed by the researcher from the premise that learning to convert between units of measurement is critical to learners development in the realm of science and other courses and that having access to a general method would support students' efficiency in conversion (Butterfield, Sutherland, \& Molyneux-Hodgson, 2000). The focus for designing the Reverse DTML-Conversion model was on the role of a general rule for converting and this arose out of a detailed observational study of first year diploma in basic education students of Jasikan College of Education working through their integrated science course on measurement (Institute of Education, 2010).

Conversions are an integral part of much scientific practice, for example to allow for ease of data processing, to enable comparison and standardization and to support the understanding of physical quantities and processes (Molyneux \& Sutherland, 1996). It is therefore crucial for students to become competent in converting between units. The researcher having interacted with students' of teacher training college and has taught for six years in Jasikan College of Education, designed a model (i.e. Reverse DTML-Conversion model) that would make the teaching of the subject practical and real to both students and tutors in Colleges of Education in Ghana. The Reverse DTML-Conversion model is much more an improved form of the metric mania and the stair-step method (Trimpe, 2008; http://www.bayhicoach.com). The Reverse DTML-Conversion model has been tested on first year students of Jasikan College of Education. First year students of the college were used because
measurement was part of their Integrated Science Syllabus.
The Reverse DTML-Conversion Model is made up of three most basic fundamental physical quantities i.e. Distance, Time and Mass. This version instead of presenting the research aspect of the reverse DTML-Conversion models rather presents only the conceptual framework and practical aspects of the Reverse DTML-Conversion model.

### 1.1 Research hypothesis

The following research hypothesis guided the researcher in transforming the DTML-Conversion model (Kumassah, 2012) into the Reverse DTML-Conversion model.
$>$ There is no significant statistical deference between the conversion-factor on length (one dimension distance) and the L-Conversion model on length (one dimension distance)
$>$ There is no significant statistical deference between the conversion-factor on area (two dimension distance) and the A-Conversion model on area (two dimension distance)
$>$ There is no significant statistical deference between the conversion-factor on volume (three dimension distance) and the V-Conversion model on volume (three dimension distance)
$>$ There is no significant statistical deference between the conversion-factor on mass and the M-Conversion model on mass
$>$ There is no significant statistical deference between the conversion factor on time and the T-Conversion model on time.
$>$ There is no significant statistical deference between the conversion factor on litre and the L-Conversion model on litre.

## 2. Conceptual framework

### 2.1 The reverse distance-conversion model

The reverse distance-Conversion Model (the reverse D-Conversion Model) has three parts, which are in three distance dimensions (the reverse Length-Conversion Model i.e. D1/L-Conversion model) which is a one dimension distance, two dimension distance (the Area-Conversion Model i.e. D2/A-Conversion model) and three dimension distance (the Volume-Conversion Model i.e. D3/V-Conversion model).

A straight line is divided into six equal parts. The distance between each part is ten (10), hundred (100), and thousand (1000) depending on one's dimension.

Figure 1. The division of the straight line into equal parts of 10, 100, and 1000
Figure 1a


Figure $1 b$


Figure 1c


The straight lines are labelled $\left(\mathrm{mm}, \mathrm{mm}^{2}, \mathrm{~mm}^{3}\right),\left(\mathrm{cm}, \mathrm{cm}^{2}, \mathrm{~cm}^{3}\right),\left(\mathrm{dm}, \mathrm{dm}^{2}, \mathrm{dm}^{3}\right),\left(\mathrm{m}, \mathrm{m}^{2}, \mathrm{~m}^{3}\right),\left(\mathrm{Dm}, \mathrm{Dm}^{2}\right.$, $\mathrm{Dm}^{3}$ ), ( $\mathrm{Hm}, \mathrm{Hm}^{2}, \mathrm{Hm}^{3}$ ), and ( $\mathrm{Km}, \mathrm{Km}^{2}, \mathrm{Km}^{3}$ ).

| $\mathrm{mm}=$ millimetre | $\mathrm{mm}^{2}=$ square millimetre | $\mathrm{mm}^{3}=$ cubic millimetre |
| :--- | :--- | :--- |
| $\mathrm{cm}=$ centimetre | $\mathrm{cm}^{2}=$ square centimetre | $\mathrm{cm}^{3}=$ cubic centimetre |
| $\mathrm{dm}=$ decimetre | $\mathrm{dm}^{2}=$ square decimetre | $\mathrm{dm}^{3}=$ cubic decimetre |
| $\mathrm{m}=$ metre | $\mathrm{m}^{2}=$ square metre | $\mathrm{m}=$ cubic metre |
| $\mathrm{Dm}=$ decametre | $\mathrm{Dm}^{2}=$ square decametre | $\mathrm{Dm}^{3}=$ cubic decametre |
| $\mathrm{Hm}=$ hectometre | $\mathrm{Hm}^{2}=$ square hectometre | $\mathrm{Hm}^{3}=$ cubic hectometre |
| $\mathrm{Km}=$ kilometre | $\mathrm{Km}^{2}=$ square kilometre | $\mathrm{Km}^{3}=$ cubic kilometer |

Figure 2. The transformation of figure-1 into complete One, Two and Three dimensions reverse distance-conversion model

Figure $2 a$


Figure $2 b$


Figure 2c


The distance-conversion model has three dimensions (i.e. One Dimension Distance, Two Dimension Distance (Area) and Three Dimension Distance (Volume)). Each dimension of distance has six steps/movements.

One dimension distance (the reverse length (l)-conversion model)

This only looks at a one space with no common meeting point between two objects i.e. Km to $\mathrm{Hm}, \mathrm{Hm}$ to

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$\mathrm{Dm}, \mathrm{Dm}$ to $\mathrm{m}, \mathrm{m}$ to $\mathrm{dm}, \mathrm{dm}$ to cm and cm to mm . from figure 1 , the straight line has been divided into 10 equal distances starting from Km to mm . This meant that 1 step of Km will give 10 steps of Hm and it follows through to mm .

NB: when dealing with conversion, then one must be in the realms of multiplication (i.e. movement from the maximum unit to the least unit) and division (i.e. movement from the least unit to the maximum unit). With this straight line on distance, the least unit is the millimetre $(\mathrm{mm})$ and the maximum unit is the kilometre $(\mathrm{Km})$.

Figure 2d


Figure 3: Complete labeled one dimension reverse distance-conversion model


Figure 3a: One Step/ Movement


In figure 3 a , one step was made starting from Km to the right i.e. the step from Km to Hm , it then implies that the first movement from Km to Hm is 10 and the second movement from Hm to Dm is also 10 , thus $10=101=10$ (i.e. $1 \mathrm{Km}=10 \mathrm{Hm}$ )

## Step One / Movement One

$1 \mathrm{Km}=10\left(1 \times 10^{1}\right) \mathrm{Hm}$
$1 \mathrm{Hm}=10\left(1 \times 10^{1}\right) \mathrm{Dm}$
$1 \mathrm{Dm}=10\left(1 \times 10^{1}\right) \mathrm{m}$
$1 \mathrm{~m}=10\left(1 \times 10^{1}\right) \mathrm{dm}$
$1 \mathrm{dm}=10\left(1 \times 10^{1}\right) \mathrm{cm}$
$1 \mathrm{~cm}=10\left(1 \times 10^{1}\right) \mathrm{mm}$

Figure 3b: Two Steps/ Movements


In figure 3 b , two steps were made starting from Km to the right i.e. the first step from Km to Hm and from Hm to Dm , it then implies that the first movement from Km to Hm is 10 and the second movement from Hm to Dm is also 10 , thus $10 \times 10=10^{2}=100$ (i.e. $1 \mathrm{Km}=100 \mathrm{Dm}$ ).

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Step Two/ Movement Two
$1 \mathrm{Km}=100\left(1 \times 10^{2}\right) \mathrm{Dm}$
$1 \mathrm{Hm}=100\left(1 \times 10^{2}\right) \mathrm{m}$
$1 \mathrm{Dm}=100\left(1 \times 10^{2}\right) \mathrm{dm}$
$1 \mathrm{~m}=100\left(1 \times 10^{2}\right) \mathrm{cm}$
$1 \mathrm{dm}=100\left(1 \times 10^{2}\right) \mathrm{mm}$

Figure 3c: Three Steps/ Movements


In figure 3 c , three steps were made starting from Km to the right i.e. the first step from Km to $\mathrm{Hm}, \mathrm{Hm}$ to Dm and from Dm to m , it then implies that the first movement from Km to Hm is 10 , the second movement from Hm to Dm is 10 and the last step from Dm to m is also 10 , thus $10 \times 10 \times 10=10^{3}=1000$ (i.e. $1 \mathrm{Km}=1000 \mathrm{~m}$ ).

## Step Threel Movement Three <br> $1 \mathrm{Km}=1000\left(1 \times 10^{3}\right) \mathrm{m}$ <br> $1 \mathrm{Hm}=1000\left(1 \times 10^{3}\right) \mathrm{dm}$ <br> $1 \mathrm{Dm}=1000\left(1 \times 10^{3}\right) \mathrm{cm}$ <br> $1 \mathrm{~m}=1000\left(1 \times 10^{3}\right) \mathrm{mm}$

Figure 3d: Four Steps/ Movements


In figure 3 d , four steps were made starting from Km to the right i.e. the first step from Km to $\mathrm{Hm}, \mathrm{Hm}$ to Dm , Dm to m and from m to dm , it then implies that the first movement from Km to Hm is 10 , the second movement from Hm to Dm is 10 , the third movement from Dm to m is 10 and the last step from m to dm is also 10 , thus $10 \times 10 \times 10 \times 10=10^{4}=10000$ (i.e. $1 \mathrm{Km}=10000 \mathrm{dm}$ ).

Step Four/ Movement Four
$1 \mathrm{Km}=10000\left(1 \times 10^{4}\right) \mathrm{dm}$
$1 \mathrm{Hm}=10000\left(1 \times 10^{4}\right) \mathrm{cm}$
$1 \mathrm{Dm}=10000\left(1 \times 10^{4}\right) \mathrm{mm}$
Figure 3e: Five Steps/ Movements


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In figure 3 e , five steps were made starting from Km to the right i.e. the first step from Km to $\mathrm{Hm}, \mathrm{Hm}$ to $\mathrm{Dm}, \mathrm{Dm}$ to $\mathrm{m}, \mathrm{m}$ to dm and from dm to cm , it then implies that the first movement from Km to Hm is 10 , the second movement from Hm to Dm is 10 , the third movement from Dm to m is 10 , the fourth movement from m to dm is 10 and the last step from dm to cm is also 10 , thus $10 \times 10 \times 10 \times 10 \times 10=10^{5}=100000$ (i.e. 1 Km $=100000 \mathrm{~cm}$ ).

## Step Fivel Movement Five <br> $1 \mathrm{Km}=100000\left(1 \times 10^{5}\right) \mathrm{cm}$ <br> $1 \mathrm{Hm}=100000\left(1 \times 10^{5}\right) \mathrm{mm}$

Figure 3f: Six Steps/ Movements


In figure 3 f , six steps were made starting from Km right i.e. the first step from Km to $\mathrm{Hm}, \mathrm{Hm}$ to $\mathrm{Dm}, \mathrm{Dm}$ to $\mathrm{m}, \mathrm{m}$ to dm , dm to cm and from cm to mm , it then implies that the first movement from Km to Hm is 10 , the second movement from Hm to Dm is 10 , the third movement from Dm to m is 10 , the fourth movement from m to dm is 10 , the fifth movement from dm to cm is 10 and the last step from cm to mm is also 10 , thus $10 \times 10 \times 10 \times 10 \times 10 \times 10=10^{6}=1000000$ (i.e. $1 \mathrm{Km}=1000000 \mathrm{~mm}$ ).

Step Six/ Movement Six
$1 \mathrm{Km}=1000000(1 \times 106) \mathrm{mm}$

Table 1
One Dimension reverse Distance Summary

| Step 1 | Step 2 | Step 3 | Step 4 | Step 5 | Step 6 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $1 \mathrm{Km}=10\left(1 \times 10^{1}\right) \mathrm{Hm}$ | $\begin{aligned} & 1 \mathrm{Km}=100\left(1 \times 10^{2}\right) \\ & \mathrm{Dm} \end{aligned}$ | $1 \mathrm{Km}=1000\left(1 \times 10^{3}\right) \mathrm{m}$ | $1 \mathrm{Km}=10000\left(1 \times 10^{4}\right) \mathrm{dm}$ | $\begin{aligned} & 1 \mathrm{Km}=100000\left(1 \times 10^{5}\right) \\ & \mathrm{cm} \end{aligned}$ | $\begin{aligned} & 1 \mathrm{Km} \\ & =1000000(1 \times \\ & \left.10^{6}\right) \mathrm{mm} \end{aligned}$ |
| $1 \mathrm{Hm}=10\left(1 \times 10^{1}\right) \mathrm{Dm}$ | $\begin{aligned} & 1 \mathrm{Hm}=100\left(1 \times 10^{2}\right) \\ & \mathrm{m} \end{aligned}$ | $1 \mathrm{Hm}=1000\left(1 \times 10^{3}\right) \mathrm{dm}$ | $1 \mathrm{Hm}=10000\left(1 \times 10^{4}\right) \mathrm{cm}$ | $\begin{aligned} & 1 \mathrm{Hm}=100000\left(1 \times 10^{5}\right) \\ & \mathrm{mm} \end{aligned}$ |  |
| $1 \mathrm{Dm}=10\left(1 \times 10^{1}\right) \mathrm{m}$ | $\begin{aligned} & 1 \mathrm{Dm}=100\left(1 \times 10^{2}\right) \\ & \mathrm{dm} \end{aligned}$ | $1 \mathrm{Dm}=1000\left(1 \times 10^{3}\right) \mathrm{cm}$ | $1 \mathrm{Dm}=10000\left(1 \times 10^{4}\right) \mathrm{mm}$ |  |  |
| $1 \mathrm{~m}=10\left(1 \times 10^{1}\right) \mathrm{dm}$ | $1 \mathrm{~m}=100\left(1 \times 10^{2}\right) \mathrm{cm}$ | $1 \mathrm{~m}=1000\left(1 \times 10^{3}\right) \mathrm{mm}$ |  |  |  |
| Table 1 continued |  |  |  |  |  |
| Step 1 | Step 2 | Step 3 | Step $4 \quad$ Step 5 | Step 6 |  |
| $1 \mathrm{dm}=10\left(1 \times 10^{1}\right) \mathrm{cm}$ | $1 \mathrm{dm}=100\left(1 \times 10^{2}\right)$ mm |  |  |  |  |
| $1 \mathrm{~cm}=10\left(1 \times 10^{1}\right) \mathrm{mm}$ |  |  |  |  |  |

### 2.2 The Reverse Two Dimension Distance model (Area (A)-Conversion Model)

Figure 4: A straight line is divided into seven equal parts. The distance between each part is hundred (100).


The number line is labelled $\mathrm{Km} 2, \mathrm{Hm} 2, \mathrm{Dm} 2, \mathrm{~m} 2, \mathrm{dm} 2, \mathrm{~cm} 2$ and mm 2 .
$\mathrm{Km} 2=$ square kilometre
$\mathrm{Hm} 2=$ square hectometre
Dm2 $=$ square decametre
$\mathrm{m} 2=$ square metre
$\mathrm{dm} 2=$ square decimetre
$\mathrm{cm} 2=$ square centimetre
$\mathrm{mm} 2=$ square millimeter

Figure 4.1: the Complete Labelled Two Dimension reverse Distance-Conversion Model (A-Conversion Model)


The two dimension of distance looks at two spaces ( ) with a common meeting point between two objects (Asiedu \& Baah-Yeboah, 2004; Brown, 1999; Serway \& Jewett, 2004; Awe \& Okunola, 1992; http://www.bayhicoach.com) i.e. $\mathrm{Km}^{2}$ to $\mathrm{Hm}^{2}, \mathrm{Hm}^{2}$ to $\mathrm{Dm}^{2}, \mathrm{Dm}^{2}$ to $\mathrm{m}^{2}, \mathrm{~m}^{2}$ to $\mathrm{dm}^{2}, \mathrm{dm}^{2}$ to $\mathrm{cm}^{2}$ and $\mathrm{cm}^{2}$ to $\mathrm{mm}^{2}$. From figure 13, a straight line has been divided into 100 equal distances starting from $\mathrm{Km}^{2}$ to $\mathrm{mm}^{2}$. This meant that 1 step of $\mathrm{Km}^{2}$ will give 100 steps of $\mathrm{Hm}^{2}$ and it follows through to $\mathrm{mm}^{2}$.

Figure 4a: One Step/ Movement


In figure 4 a , one step was made starting from $\mathrm{Km}^{2}$ right i.e. the step from $\mathrm{Km}^{2}$ to $\mathrm{Hm}^{2}$, it then implies that the first movement from $\mathrm{Km}^{2}$ to $\mathrm{Hm}^{2}$ is 100 and the second movement from $\mathrm{Hm}^{2}$ to $\mathrm{Dm}^{2}$ is also 100 , thus $100=10^{2}=100$ (i.e. $1 \mathrm{Km}^{2}=100 \mathrm{Hm} 2$ ).

The Mathematical proof:
Area= length ${ }^{*}$ length $=L^{2}$
$1 \mathrm{Km}^{2}=1 \mathrm{Km} \times 1 \mathrm{Km}$
But $1 \mathrm{Km}=10 \mathrm{Hm}$
$\Longrightarrow 1 \mathrm{Km}^{2}=1 \mathrm{Km} \times 1 \mathrm{Km}=10 \mathrm{Hm} \times 10 \mathrm{Hm}=10 \times 10(\mathrm{Hm} \times \mathrm{Hm})=10^{2} \mathrm{Hm}^{2}=100 \mathrm{Hm}^{2}$

## Step One / Movement One

$1 \mathrm{Km}^{2}=100\left(1 \times 10^{2}\right) \mathrm{Hm}^{2}$
$1 \mathrm{Hm}^{2}=100\left(1 \times 10^{2}\right) \mathrm{Dm}^{2}$
$1 \mathrm{Dm}^{2}=100\left(1 \times 10^{2}\right) \mathrm{m}^{2}$
$1 \mathrm{~m}^{2}=100\left(1 \times 10^{2}\right) \mathrm{dm}^{2}$
$1 \mathrm{dm}^{2}=100\left(1 \times 10^{2}\right) \mathrm{cm}^{2}$
$2.31 \mathrm{~cm}^{2}=100\left(1 \times 10^{2}\right) \mathrm{mm}^{2}$

Figure 4b: Two Step/ Movement


In figure 4 b , two steps were made starting from $\mathrm{Km}^{2}$ right i.e. the first step from $\mathrm{Km}^{2}$ to $\mathrm{Hm}^{2}$ and from $\mathrm{Hm}^{2}$ to $\mathrm{Dm}^{2}$. It then implies that the first movement from $\mathrm{Km}^{2}$ to $\mathrm{Hm}^{2}$ is 100 and the second movement from $\mathrm{Hm}^{2}$ to $\mathrm{Dm}^{2}$ is also 100 , thus $100 \times 100=10^{4}=10000$ (i.e. $1 \mathrm{Km}^{2}=10000 \mathrm{Dm}^{2}$ ).

> Step Two/ Movement Two
> $1 \mathrm{Km}^{2}=10000\left(1 \times 10^{4}\right) \mathrm{Dm}^{2}$
> $1 \mathrm{Hm}^{2}=10000\left(1 \times 10^{4}\right) \mathrm{m}^{2}$
> $1 \mathrm{Dm}^{2}=10000\left(1 \times 10^{4}\right) \mathrm{dm}^{2}$
> $1 \mathrm{~m}^{2}=10000\left(1 \times 10^{4}\right) \mathrm{cm}^{2}$
> $1 \mathrm{dm}^{2}=10000\left(1 \times 10^{4}\right) \mathrm{mm}^{2}$

Figure 4c: Three Steps/ Movements


In figure 4 c , three steps were made starting from $\mathrm{Km}^{2}$ to the right i.e. the first step from $\mathrm{Km}^{2}$ to $\mathrm{Hm}^{2}, \mathrm{Hm}^{2}$ to $\mathrm{Dm}^{2}$ and from $\mathrm{Dm}^{2}$ to $\mathrm{m}^{2}$, it then implies that the first movement from $\mathrm{Km}^{2}$ to $\mathrm{Hm}^{2}$ is 100 , the second movement from $\mathrm{Hm}^{2}$ to $\mathrm{Dm}^{2}$ is 100 and the last step from $\mathrm{Dm}^{2}$ to $\mathrm{m}^{2}$ is also 100 , thus $100 \times 100 \times 100=10^{6}=1000000$ (i.e. $1 \mathrm{~km}^{2}=1000000 \mathrm{~m}^{2}$ ).

## Step Threel Movement Three

$1 \mathrm{Km}^{2}=1000000\left(1 \times 10^{6}\right) \mathrm{m}^{2}$
$1 \mathrm{Hm}^{2}=1000000\left(1 \times 10^{6}\right) \mathrm{dm}^{2}$
$1 \mathrm{Dm}^{2}=1000000\left(1 \times 10^{6}\right) \mathrm{cm}^{2}$
$1 \mathrm{~m}^{2}=1000000\left(1 \times 10^{6}\right) \mathrm{mm}^{2}$

Figure 4d: Four Steps/ Movements


In figure 4 d , four steps were made starting from $\mathrm{Km}^{2}$ to the right i.e. the first step from $\mathrm{Km}^{2}$ to $\mathrm{Hm}^{2}, \mathrm{Hm}^{2}$ to $\mathrm{Dm}^{2}, \mathrm{Dm}^{2}$ to $\mathrm{m}^{2}$ and from $\mathrm{m}^{2}$ to $\mathrm{dm}^{2}$, it then implies that the first movement from $\mathrm{Km}^{2}$ to $\mathrm{Hm}^{2}$ is 100 , the second movement from $\mathrm{Hm}^{2}$ to $\mathrm{Dm}^{2}$ is $100, \mathrm{Dm}^{2}$ to $\mathrm{m}^{2}$ is 100 and the last step from $\mathrm{m}^{2}$ to $\mathrm{dm}^{2}$ is also 100 , thus $100 \times 100 \times 100 \times 100=10^{8}=100000000$ (i.e. $1 \mathrm{Km}^{2}=100000000 \mathrm{dm}^{2}$ )

## Step Four/ Movement Four

$1 \mathrm{Km}^{2}=100000000\left(1 \times 10^{8}\right) \mathrm{dm}^{2}$
$1 \mathrm{Hm}^{2}=100000000\left(1 \times 10^{8}\right) \mathrm{cm}^{2}$
$1 \mathrm{Dm}^{2}=100000000\left(1 \times 10^{8}\right) \mathrm{mm}^{2}$

Figure 4e: Five Steps/ Movements


In figure 4 e , five steps were made starting from $\mathrm{Km}^{2}$ to the right i.e. the first step from $\mathrm{Km}^{2}$ to $\mathrm{Hm}^{2}, \mathrm{Hm}^{2}$ to $\mathrm{Dm}^{2}, \mathrm{Dm}^{2}$ to $\mathrm{m}^{2}, \mathrm{~m}^{2}$ to $\mathrm{dm}^{2}$ and from $\mathrm{dm}^{2}$ to $\mathrm{cm}^{2}$, it then implies that the first movement from $\mathrm{Km}^{2}$ to $\mathrm{Hm}^{2}$ is 100 , the second movement from $\mathrm{Hm}^{2}$ to $\mathrm{Dm}^{2}$ is $100, \mathrm{Dm}^{2}$ to $\mathrm{m}^{2}$ is $100, \mathrm{~m}^{2}$ to $\mathrm{dm}^{2}$ and the last step from $\mathrm{dm}^{2}$ to $\mathrm{cm}^{2}$ is also 100 , thus $100 \times 100 \times 100 \times 100 \times 100=10^{10}=10000000000$ (i.e. $1 \mathrm{Km}^{2}=10000000000 \mathrm{~cm}^{2}$ ).

## Step Five/ Movement Five

$$
\begin{aligned}
& 1 \mathrm{Km}^{2}=10000000000\left(1 \times 10^{10}\right) \mathrm{cm}^{2} \\
& 1 \mathrm{Hm}^{2}=10000000000\left(1 \times 10^{10}\right) \mathrm{mm}^{2}
\end{aligned}
$$

## Figure 4f: Six Steps/ Movements



In figure 4 f , six steps were made starting from $\mathrm{Km}^{2}$ to the right i.e. the first step from $\mathrm{Km}^{2}$ to $\mathrm{Hm}^{2}, \mathrm{Hm}^{2}$ to $\mathrm{Dm}^{2}, \mathrm{Dm}^{2}$ to $\mathrm{m}^{2}, \mathrm{~m}^{2}$ to $\mathrm{dm}^{2}, \mathrm{dm}^{2}$ to $\mathrm{cm}^{2}$ and from $\mathrm{cm}^{2}$ to $\mathrm{mm}^{2}$, it then implies that the first movement from $\mathrm{Km}^{2}$ to $\mathrm{Hm}^{2}$ is 100 , the second movement from $\mathrm{Hm}^{2}$ to $\mathrm{Dm}^{2}$ is $100, \mathrm{Dm}^{2}$ to $\mathrm{m}^{2}$ is $100, \mathrm{~m}^{2}$ to $\mathrm{dm}^{2}, \mathrm{dm}^{2}$ to $\mathrm{cm}^{2}$ and the last step from $\mathrm{cm}^{2}$ to $\mathrm{mm}^{2}$ is also 100 , thus $100 \times 100 \times 100 \times 100 \times 100 \times 100=10^{12}=1000000000000$ (i.e. $1 \mathrm{Km}^{2}=10000000000 \mathrm{~mm}^{2}$ ).

Step Six/ Movement Six
$1 \mathrm{Km}^{2}=10000000000\left(1 \times 10^{12}\right) \mathrm{mm}^{2}$

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Table 2
Two Dimension Distance Summary

| Step 1 | Step 2 | Step 3 | Step 4 | Step 5 | Step 6 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $1 \mathrm{Km}^{2}=100\left(1 \times 10^{2}\right) \mathrm{Hm}^{2}$ | $1 \mathrm{Km}^{2}=10000(1 \times 1$ | $1 \mathrm{Km}^{2}=1000000(1 \times 1$ | $1 \mathrm{Km}^{2}=100000000(1 \times 1$ | $1 \mathrm{Km}^{2}=10000000000$ | $1 \mathrm{Km}^{2}=10000000000$ |
|  | $\left.0^{4}\right) \mathrm{Dm}^{2}$ | $\left.0^{6}\right) \mathrm{m}^{2}$ | $\left.0^{8}\right) \mathrm{dm}^{2}$ | $\left(1 \times 10^{10}\right) \mathrm{cm}^{2}$ | $\left(1 \times 10^{12}\right) \mathrm{mm}^{2}$ |
| $1 \mathrm{Hm}^{2}=100\left(1 \times 10^{2}\right) \mathrm{Dm}^{2}$ | $1 \mathrm{Hm}^{2}=10000(1 \times 1$ | $1 \mathrm{Hm}^{2}=1000000(1 \times 1$ | $1 \mathrm{Hm}^{2}=100000000(1 \times 1$ | $1 \mathrm{Hm}^{2}=10000000000$ |  |
|  | $\left.0^{4}\right) \mathrm{m}^{2}$ | $\left.0^{6}\right) \mathrm{dm}^{2}$ | $\left.0^{8}\right) \mathrm{cm}^{2}$ | $\left(1 \times 10^{10}\right) \mathrm{mm}^{2}$ |  |
| $1 \mathrm{Dm}^{2}=100\left(1 \times 10^{2}\right) \mathrm{m}^{2}$ | $1 \mathrm{Dm}^{2}=10000(1 \times 1$ | $1 \mathrm{Dm}^{2}=1000000(1 \times 1$ | $1 \mathrm{Dm}^{2}=100000000(1 \times 1$ |  |  |
|  | $\left.0^{4}\right) \mathrm{dm}^{2}$ | $\left.0^{6}\right) \mathrm{cm}^{2}$ | $\left.0^{8}\right) \mathrm{mm}^{2}$ |  |  |
| $1 \mathrm{~m}^{2}=100\left(1 \times 10^{2}\right) \mathrm{dm}^{2}$ | $1 \mathrm{~m}^{2}=10000(1 \times 10$ | $1 \mathrm{~m}^{2}=1000000\left(1 \times 10^{6}\right.$ |  |  |  |
| $1 \mathrm{dm}^{2}=100\left(1 \times 10^{2}\right) \mathrm{cm}^{2}$ | $4) \mathrm{cm}^{2}$ | $1 \mathrm{dm}^{2}=10000(1 \times 1$ |  | $\mathrm{mm}^{2}$ |  |
|  | $\left.0^{4}\right) \mathrm{mm}^{2}$ |  |  |  |  |
| $1 \mathrm{~cm}^{2}=100\left(1 \times 10^{2}\right) \mathrm{mm}^{2}$ |  |  |  |  |  |

### 2.4 Reverse Three Dimension Distance (Volume (V)-Conversion Model)

Figure 5: A straight line is divided into seven equal parts. The distance between each part is one thousand (1000)


The number line is labelled $\mathrm{Km}^{3}, \mathrm{Hm}^{3}, \mathrm{Dm}^{3}, \mathrm{~m}^{3}, \mathrm{dm}^{3}, \mathrm{~cm}^{3}$ and $\mathrm{mm}^{3}$.
$\mathrm{Km}^{3}=$ cubic kilometre
$\mathrm{Hm}^{3}=$ cubic hectometre
$\mathrm{Dm}^{3}=$ cubic decametre
$\mathrm{m}^{3}=$ cubic metre
$\mathrm{dm}^{3}=$ cubic decimetre
$\mathrm{cm}^{3}=$ cubic centimetre
$\mathrm{mm}^{3}=$ cubic millimeter

Figure 5.1: The Complete Labelled Three Dimension Reverse Distance-Conversion Model (V-Conversion Model)


The three dimension of distance looks at three spaces (—) with a common meeting point between two objects (Asiedu \& Baah-Yeboah, 2004; Brown, 1999; Serway \& Jewett, 2004; Awe \& Okunola, 1992; http://www.bayhicoach.com) i.e. $\mathrm{Km}^{3}$ to $\mathrm{Hm}^{3}, \mathrm{Hm}^{3}$ to $\mathrm{Dm}^{3}, \mathrm{Dm}^{3}$ to $\mathrm{m}^{3}, \mathrm{~m}^{3}$ to $\mathrm{dm}^{3}, \mathrm{dm}^{3}$ to $\mathrm{cm}^{3}$ and $\mathrm{cm}^{3}$ to $\mathrm{mm}^{3}$. From figure 20, a straight line has been divided into 1000 equal distances starting from Km 3 to mm 3 . This meant that 1 step of $\mathrm{Km}^{3}$ will give 1000 steps of $\mathrm{Hm}^{3}$ and it follows through to mm3.

## Figure 5a: One Step/ Movement



In figure 5 a , one step was made starting from $\mathrm{Km}^{3}$ to the right i.e. the step from $\mathrm{Km}^{3}$ to $\mathrm{Hm}^{3}$ is 1000 , and the movement from $\mathrm{Hm}^{3}$ to $\mathrm{Dm}^{3}$ is also 1000 , thus $1 \mathrm{~km}^{3}=1000\left(10^{3}\right) \mathrm{Hm}^{3}$. The Mathematical proof:

Volume $=$ length $\times$ length $\times$ Length $=L^{3}$
$1 \mathrm{Km}^{3}=1 \mathrm{Km} \times 1 \mathrm{Km} \times 1 \mathrm{Km}$
But $1 \mathrm{Km}=10 \mathrm{Hm}$
$\Rightarrow 1 \mathrm{Km}^{3}=1 \mathrm{Km} \times 1 \mathrm{Km} \times 1 \mathrm{Km} \equiv 10 \mathrm{Hm} \times 10 \mathrm{Hm} \times 10 \mathrm{Hm}=(10)^{3}(\mathrm{Hm})^{3}=1000 \mathrm{Hm}^{3}$

Step Onel Movement One
$1 \mathrm{Km}^{3}=1000\left(10^{3}\right) \mathrm{Hm}^{3}$
$1 \mathrm{Hm}^{3}=1000\left(10^{3}\right) \mathrm{Dm}^{3}$
$1 \mathrm{Dm}^{3}=1000\left(10^{3}\right) \mathrm{m}^{3}$
$1 \mathrm{~m}^{3}=1000\left(10^{3}\right) \mathrm{dm}^{3}$
$1 \mathrm{dm}^{3}=1000\left(10^{3}\right) \mathrm{cm}^{3}$
$1 \mathrm{~cm}^{3}=1000\left(10^{3}\right) \mathrm{mm}^{3}$

Figure 5b: Two Step/ Movement


In figure 5 b, two steps were made starting from $\mathrm{Km}^{3}$ to the right i.e. the first step from $\mathrm{Km}^{3}$ to $\mathrm{Hm}^{3}$ and from $\mathrm{Hm}^{3}$ to $\mathrm{Dm}^{3}$. It then implies that one movement from $\mathrm{Km}^{3}$ to $\mathrm{Hm}^{3}$ is 1000 and the second movement from $\mathrm{Hm}^{3}$ to $\mathrm{Dm}^{3}$ is also 1000 , thus $1000 \times 1000=10^{6}=1000000$ (i.e. $1 \mathrm{Km}^{3}=1000000 \mathrm{Dm}^{3}$ ).

Step Two/ Movement Two
$1 \mathrm{Km}^{3}=1000000\left(1 \times 10^{6}\right) \mathrm{Dm}^{3}$
$1 \mathrm{Hm}^{3}=1000000\left(1 \times 10^{6}\right) \mathrm{m}^{3}$
$1 \mathrm{Dm}^{3}=1000000\left(1 \times 10^{6}\right) \mathrm{dm}^{3}$
$1 \mathrm{~m}^{3}=1000000\left(1 \times 10^{6}\right) \mathrm{cm}^{3}$
$1 \mathrm{dm}^{3}=1000000\left(1 \times 10^{6}\right) \mathrm{mm}^{3}$

Figure 5c: Three Steps/ Movements


In figure 5 c , three steps were made starting from $\mathrm{Km}^{3}$ to the right i.e. the first step from $\mathrm{Km}^{3}$ to $\mathrm{Hm}^{3}, \mathrm{Hm}^{3}$ to $\mathrm{Dm}^{3}$ and from $\mathrm{Dm}^{3}$ to $\mathrm{m}^{3}$, it then implies that the first movement from $\mathrm{Km}^{3}$ to $\mathrm{Hm}^{3}$ is 1000 , the second movement from $\mathrm{Hm}^{3}$ to $\mathrm{Dm}^{3}$ is 1000 and the last step from $\mathrm{Dm}^{3}$ to $\mathrm{m}^{3}$ is also 1000 , thus $1000 \times 1000 \times 1000=10^{9}=1000000000$ (i.e. $1 \mathrm{Km}^{3}=1000000000 \mathrm{~m}^{3}$ ).

Step Threel Movement Three

$$
\begin{aligned}
& 1 \mathrm{Km}^{3}=1000000000\left(1 \times 10^{9}\right) \mathrm{m}^{3} \\
& 1 \mathrm{Hm}^{3}=1000000000\left(1 \times 10^{9}\right) \mathrm{dm}^{3} \\
& 1 \mathrm{Dm}^{3}=1000000000\left(1 \times 10^{9}\right) \mathrm{cm}^{3} \\
& 1 \mathrm{~m}^{3}=1000000000\left(1 \times 10^{9}\right) \mathrm{mm}^{3}
\end{aligned}
$$

Figure 5d: Four Steps/ Movements


In figure 5 d , four steps were made starting from $\mathrm{Km}^{3}$ to the right i.e. the first step from $\mathrm{Km}^{3}$ to $\mathrm{Hm}^{3}, \mathrm{Hm}^{3}$ to $\mathrm{Dm}^{3}, \mathrm{Dm}^{3}$ to $\mathrm{m}^{3}$ and from $\mathrm{m}^{3}$ to $\mathrm{dm}^{3}$, it then implies that the first movement from $\mathrm{Km}^{3}$ to $\mathrm{Hm}^{3}$ is 1000 , the second movement from $\mathrm{Hm}^{3}$ to $\mathrm{Dm}^{3}$ is $1000, \mathrm{Dm}^{3}$ to $\mathrm{m}^{3}$ is 1000 and the last step from $\mathrm{m}^{3}$ to $\mathrm{dm}^{3}$ is 1000 , thus $1000 \times 1000 \times 1000 \times 1000=10^{12}=1000000000000$ (i.e. $1 \mathrm{Km}^{3}=1000000000000 \mathrm{dm}^{3}$ ).

> Step Four/ Movement Four
> $1 \mathrm{Km}^{3}=1000000000000\left(1 \times 10^{12}\right) \mathrm{dm}^{3}$
> $1 \mathrm{Hm}^{3}=1000000000000\left(1 \times 10^{12}\right) \mathrm{cm}^{3}$
> $1 \mathrm{Dm}^{3}=1000000000000\left(1 \times 10^{12}\right) \mathrm{mm}^{3}$

Figure 5e: Five Steps/ Movements


In figure 5 e, five steps were made starting from $\mathrm{Km}^{3}$ to the right i.e. the first step from $\mathrm{Km}^{3}$ to $\mathrm{Hm}^{3}, \mathrm{Hm}^{3}$ to $\mathrm{Dm}^{3}, \mathrm{Dm}^{3}$ to $\mathrm{m}^{3}, \mathrm{~m}^{3}$ to $\mathrm{dm}^{3}$ and from $\mathrm{dm}^{3}$ to $\mathrm{cm}^{3}$, it then implies that the first movement from $\mathrm{Km}^{3}$ to $\mathrm{Hm}^{3}$ is 1000 , the second movement from $\mathrm{Hm}^{3}$ to $\mathrm{Dm}^{3}$ is 1000 , to the last step from $\mathrm{dm}^{3}$ to $\mathrm{cm}^{3}$ is also 1000 , thus $1000 \times 1000 \times 1000 \times 1000 \times 1000=10^{15}=1000000000000000$ (i.e. $1 \mathrm{Hm}^{3}=1000000000000000 \mathrm{~mm}^{3}$ ).

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> Step Five/ Movement Five
> $1 \mathrm{Km}^{3}=1000000000000000\left(1 \times 10^{15}\right) \mathrm{cm}^{3}$
> $1 \mathrm{Hm}^{3}=1000000000000000\left(1 \times 10^{15}\right) \mathrm{mm}^{3}$

Figure 5f: Six Steps/ Movements


In figure 5 f , six steps were made starting from $\mathrm{Km}^{3}$ to the right. The i.e. the steps are $\mathrm{Km}^{3}$ to $\mathrm{mm}^{3}, \mathrm{Hm}^{3}$ to $\mathrm{Dm}^{3}, \mathrm{Dm}^{3}$ to $\mathrm{m}^{3}, \mathrm{~m}^{3}$ to $\mathrm{dm}^{3}, \mathrm{dm}^{3}$ to $\mathrm{cm}^{3}$ and from $\mathrm{cm}^{3}$ to $\mathrm{mm}^{3}$, it then implies that the first movement from $\mathrm{Km}^{3}$ to $\mathrm{Hm}^{3}$ is 1000 , the second movement from $\mathrm{Hm}^{3}$ to $\mathrm{Dm}^{3}$ is $1000, \mathrm{Dm}^{3}$ to $\mathrm{m}^{3}$ is $1000, \mathrm{~m}^{3}$ to $\mathrm{dm}^{3}$ is $1000, \mathrm{dm}^{3}$ to $\mathrm{cm}^{3}$ is 1000 and the last step from $\mathrm{cm}^{3}$ to $\mathrm{mm}^{3}$ is also 1000, thus $1000 \times 1000 \times 1000 \times 1000 \times 1000 \times 1000=10^{18}=1000000000000000000\left(\right.$ i.e. $1 \mathrm{Km}^{3}=10000000000000000 \mathrm{~mm}^{3}$ ).

Step Six/ Movement Six
$1 \mathrm{Km}^{3}=10000000000000000 \mathrm{~mm}^{3}$

## Table 3

reverse Three Dimension Distance Summary

| Step 1 | Step 2 | Step 3 | Step 4 | Step 5 | Step 6 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $1 \mathrm{Km}^{3}=1000\left(10^{3}\right)$ | $1 \mathrm{Km} 3=1000000(1 \times 1$ | $1 \mathrm{Km}^{3}=1000000000$ | $1 \mathrm{Km}^{3}=10000000000$ | $1 \mathrm{Km}^{3}=10000000000$ | $1 \mathrm{Km}^{3}=10000000000$ |
| $\mathrm{Hm}^{3}$ | $06) \mathrm{Dm} 3$ | $\left(1 \times 10^{9}\right) \mathrm{m}^{3}$ | $00\left(1 \times 10^{12}\right) \mathrm{dm}^{3}$ | $00000\left(1 \times 10^{15}\right) \mathrm{cm}^{3}$ | $000000\left(1 \times 10^{18}\right) \mathrm{mm}^{3}$ |
| $1 \mathrm{Hm}^{3}=1000\left(10^{3}\right)$ | $1 \mathrm{Hm} 3=1000000(1 \times 1$ | $1 \mathrm{Hm}^{3}=1000000000($ | $1 \mathrm{Hm}^{3}=10000000000$ | $1 \mathrm{Hm}^{3}=10000000000$ |  |
| $\mathrm{Dm}^{3}$ | $06) \mathrm{m} 3$ | $\left.1 \times 10^{9}\right) \mathrm{dm}^{3}$ | $00\left(1 \times 10^{12}\right) \mathrm{cm}^{3}$ | $00000\left(1 \times 10^{15}\right) \mathrm{mm}^{3}$ |  |
| $1 \mathrm{Dm}^{3}=1000\left(10^{3}\right) \mathrm{m}^{3}$ | $1 \mathrm{Dm} 3=1000000(1 \times 1$ | $1 \mathrm{Dm}^{3}=1000000000($ | $1 \mathrm{Dm}^{3}=10000000000$ |  |  |
|  | $06) \mathrm{dm} 3$ | $\left.1 \times 10^{9}\right) \mathrm{cm}^{3}$ | $00\left(1 \times 10^{12}\right) \mathrm{mm}^{3}$ |  |  |
| $1 \mathrm{~m}^{3}=1000\left(10^{3}\right) \mathrm{dm}^{3}$ | $1 \mathrm{~m} 3=1000000(1 \times 10$ | $1 \mathrm{~m}^{3}=1000000000(1 \times$ |  |  |  |
|  | $6) \mathrm{cm} 3$ | $\left.10^{9}\right) \mathrm{mm}^{3}$ |  |  |  |
| $1 \mathrm{dm}^{3}=1000\left(10^{3}\right) \mathrm{cm}^{3}$ | $1 \mathrm{Km} 3=1000000(1 \times 1$ |  |  |  |  |
| $1 \mathrm{~cm}^{3}=1000\left(10^{3}\right)$ | $06) \mathrm{Dm} 3$ |  |  |  |  |
| $\mathrm{~mm}^{3}$ |  |  |  |  |  |

### 2.5 The Reverse Time (T)-Conversion Model

Figure 6: A number line is divided into two equal parts. The distance between each part is sixty (60)


The number line is labelled Hr , min, and s.
$\mathrm{s}=$ second

$$
\begin{aligned}
& \min =\text { minute } \\
& \mathrm{Hr}=\text { hour }
\end{aligned}
$$

Figure 6.1: The Complete Labelled Reverse T-Conversion Model


The time-conversion model has two steps (i.e. Step One, and Step Two).

Figure 6a: One Step/ Movement


In figure 6a, one step was made starting from Hr to the right i.e. the step from Hr to min, it then implies that the first movement from Hr to min is 60 and the second movement from min to s is also 60 , thus $60=6 \times 10^{1}=6 \times 10$ (i.e. $1 \mathrm{Hr}=60 \mathrm{~min}$ ).

```
Step One / Movement One
\(1 \mathrm{Hr}=60 \mathrm{~min}\)
\(1 \mathrm{~min}=60 \mathrm{~s}\)
```

Figure 6b: Two Steps/ Movements


In figure 6 b , two steps were made starting from Hr to the right i.e. the first step from Hr to min and from min to s , it then implies that the first movement from Hr to min is 60 and the second movement from min to s is also 60 , thus $60 \times 60=6^{2} \times 10^{2}=36 \times 100$ (i.e. $1 \mathrm{Hr}=3600 \mathrm{~s}$ ).

Step Two/ Movement Two
$1 \mathrm{Hr}=3600 \mathrm{~s}$

## Table 4

The Reverse Time-Conversion Model Summary

| Step 1 | Step 2 |
| :--- | :--- |
| $1 \mathrm{Hr}=60 \mathrm{~min}$ | $1 \mathrm{Hr}=3600 \mathrm{~s}$ |
| $1 \mathrm{~min}=60 \mathrm{~s}$ |  |

### 2.6 The Reverse Mass (M)-Conversion Model

Figure 7: A straight line is divided into six equal parts. The distance between each part is ten (10)


The number line is labelled $\mathrm{Kg}, \mathrm{Hg}, \mathrm{Dg}, \mathrm{g}$, dg , cg and mg .
$\mathrm{Kg}=$ kilogram
$\mathrm{Hg}=$ hectogram
Dg = dekagram
$\mathrm{g}=$ gram
dg $=$ decigram
$\mathrm{cg}=$ centigram
$\mathrm{mg}=$ milligram

Figure 7.1: The Complete Labelled Reverse M-Conversion Model


Figure 7a: One Step/ Movement


In figure 7 a , one step was made starting from Kg to the right i.e. the step from Kg to $\mathrm{Hg}, \mathrm{Hg}$ to $\mathrm{Dg}, \mathrm{Dg}$ to g , g to dg , dg to cg , and cg to mg . This then implies that one movement from Kg to Hg is 10 thus $10=1 \times 10^{1}$ (i.e. $1 \mathrm{Kg}=10 \mathrm{Hg})$.

Step One / Movement One
$1 \mathrm{Kg}=10 \mathrm{Hg}$
$1 \mathrm{Hg}=10 \mathrm{Dg}$
$1 \mathrm{Dg}=10 \mathrm{~g}$
$1 \mathrm{~g}=1 \mathrm{dg}$
$1 \mathrm{dg}=10 \mathrm{cg}$
$1 \mathrm{cg}=10 \mathrm{mg}$

Figure 7b: Two Step/ Movement


In figure 7 b , two steps were made starting from Kg to the right i.e. the step from Kg to $\mathrm{Dg}, \mathrm{Hg}$ to $\mathrm{g}, \mathrm{Dg}$ to $\mathrm{dg}, \mathrm{g}$ to cg , and dg to mg . This then implies that one movement from Kg to Dg is 100 thus $10 \times 10=1 \times 10^{2}$ (i.e. $1 \mathrm{Kg}=100 \mathrm{Dg})$.

> Step Two $/$ Movement Two
> $1 \mathrm{Kg}=100 \mathrm{Dg}$
> $1 \mathrm{Hg}=100 \mathrm{~g}$
> $1 \mathrm{Dg}=100 \mathrm{dg}$
> $1 \mathrm{~g}=100 \mathrm{cg}$
> $1 \mathrm{dg}=100 \mathrm{mg}$

Figure 7c: Three Step/ Movement


In figure 7 c , three steps were made starting from Kg to the right i.e. the step from Kg to $\mathrm{g}, \mathrm{Hg}$ to $\mathrm{dg}, \mathrm{Dg}$ to cg , and g to mg . This then implies that one movement from Kg to g is 1000 thus $10 \times 10 \times 10=1 \times 10^{3}$ (i.e. $1 \mathrm{Kg}=$ 1000 g ).

Step Three / Movement Three
$1 \mathrm{Kg}=1000 \mathrm{~g}$
$1 \mathrm{Hg}=1000 \mathrm{dg}$
$1 \mathrm{Dg}=1000 \mathrm{cg}$
$1 \mathrm{~g}=1000 \mathrm{mg}$

Figure 7d: Four Step/ Movement


In figure 7 d , four steps were made starting from Kg to the right i.e. the step from Kg to $\mathrm{dg}, \mathrm{Hg}$ to cg , and Dg to mg . This then implies that one movement from Kg to dg is 10000 thus $10 \times 10 \times 10 \times 10=1 \times 10^{4}$ (i.e. $1 \mathrm{Kg}=$ 10000 dg ).

Step Four / Movement Four
$1 \mathrm{Kg}=10000 \mathrm{dg}$
$1 \mathrm{Hg}=10000 \mathrm{cg}$
$1 \mathrm{Dg}=10000 \mathrm{mg}$

Figure 7e: Five Step/ Movement


In figure 7 e , five steps were made starting from Kg to the right i.e. the step from Kg to cg , and Hg to mg . This then implies that one movement from Kg to cg is 100000 thus $10 \times 10 \times 10 \times 10 \times 10=1 \times 10^{5}$ (i.e. $1 \mathrm{Kg}=100000$ $\mathrm{cg})$.

## Step Five / Movement Four

$$
\begin{aligned}
& 1 \mathrm{Kg}=100000 \mathrm{cg} \\
& 1 \mathrm{Hg}=100000 \mathrm{mg}
\end{aligned}
$$

Figure 7f: Six Step/ Movement


In figure 7 f , six steps were made starting from Kg to the right i.e. the step from Kg to mg . This then implies that one movement from Kg to mg is 1000000 thus $10 \times 10 \times 10 \times 10 \times 10 \times 10=1 \times 10^{6}$ (i.e. $1 \mathrm{Kg}=1000000 \mathrm{mg}$ ).

Step Six / Movement Four
$1 \mathrm{Kg}=1000000 \mathrm{mg}$

Table 5
Reverse Mass-Conversion Model Summary

| Step 1 | Step 2 | Step 3 | Step 4 | Step 5 | Step 6 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $1 \mathrm{Kg}=10 \mathrm{Hg}$ | $1 \mathrm{Kg}=100 \mathrm{Dg}$ | $1 \mathrm{Kg}=1000 \mathrm{~g}$ | $1 \mathrm{Kg}=10000 \mathrm{dg}$ | $1 \mathrm{Kg}=100000 \mathrm{cg}$ | $1 \mathrm{Kg}=1000000 \mathrm{mg}$ |
| $1 \mathrm{Hg}=10 \mathrm{Dg}$ | $1 \mathrm{Hg}=100 \mathrm{~g}$ | $1 \mathrm{Hg}=1000 \mathrm{dg}$ | $1 \mathrm{Hg}=10000 \mathrm{cg}$ | $1 \mathrm{Hg}=100000 \mathrm{mg}$ |  |
| $1 \mathrm{Dg}=10 \mathrm{~g}$ | $1 \mathrm{Dg}=100 \mathrm{dg}$ | $1 \mathrm{Dg}=1000 \mathrm{cg}$ | $1 \mathrm{Dg}=10000 \mathrm{mg}$ |  |  |
| $1 \mathrm{~g}=1 \mathrm{dg}$ | $1 \mathrm{~g}=100 \mathrm{cg}$ | $1 \mathrm{~g}=1000 \mathrm{mg}$ |  |  |  |
| $1 \mathrm{dg}=10 \mathrm{cg}$ | $1 \mathrm{dg}=100 \mathrm{mg}$ |  |  |  |  |
| $1 \mathrm{cg}=10 \mathrm{mg}$ |  |  |  |  |  |
| $1 \mathrm{dg}=10 \mathrm{cg}$ |  |  |  |  |  |
| $1 \mathrm{cg}=10 \mathrm{mg}$ |  |  |  |  |  |



### 2.7 The Reverse Liter (L)-Conversion Model

Figure 8: A straight line is divided into six equal parts. The distance between each part is ten (10)

The number line is labelled KL, $\mathrm{HL}, \mathrm{DL}, \mathrm{L}, \mathrm{dL}, \mathrm{cL}$ and mL .
KL $=$ kiloliter
HL = hectoliter
DL $=$ dekaliter
$\mathrm{L}=$ liter
$\mathrm{dL}=$ deciliter
$\mathrm{cL}=$ centiliter
$\mathrm{mL}=$ milliliter

Figure 8.1: The Complete Labelled reverse L-Conversion Model


Figure 8a: One Step/ Movement


In figure 8 a , one step was made starting from KL to the right i.e. the step from KL to $\mathrm{HL}, \mathrm{HL}$ to $\mathrm{DL}, \mathrm{DL}$ to $\mathrm{L}, \mathrm{L}$ to $\mathrm{dL}, \mathrm{dL}$ to cL , and cL to mL . This then implies that one movement from KL to HL is 10 thus $10=1 \times 10^{1}$ (i.e. $1 \mathrm{KL}=10 \mathrm{HL}$ ).

Step One / Movement One
$1 \mathrm{KL}=10 \mathrm{HL}$
$1 \mathrm{HL}=10 \mathrm{DL}$
$1 \mathrm{DL}=1 \mathrm{~L}$
$1 \mathrm{~L}=10 \mathrm{dL}$
$1 \mathrm{dL}=10 \mathrm{cL}$
$1 \mathrm{cL}=10 \mathrm{~mL}$

Figure 8b: Two Step/ Movement


In figure 8 b , two steps were made starting from KL to the right i.e. the step from KL to DL, HL to $\mathrm{L}, \mathrm{DL}$ to $\mathrm{dL}, \mathrm{L}$ to cL , and dL to mL . This then implies that the movement from KL to DL is 100 thus $10 \times 10=1 \times 10^{2}$ (i.e. $1 \mathrm{KL}=100 \mathrm{DL}$ ).

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Step Two / Movement Two
$1 \mathrm{KL}=100 \mathrm{DL}$
$1 \mathrm{HL}=100 \mathrm{~L}$
$1 \mathrm{DL}=100 \mathrm{dL}$
$1 \mathrm{~L}=1 \mathrm{cL}$
$100 \mathrm{dL}=1 \mathrm{~mL}$

Figure 8c: Three Step/ Movement


In figure 8 c , three steps were made starting from KL to the right i.e. the step from KL to $\mathrm{L}, \mathrm{HL}$ to $\mathrm{dL}, \mathrm{DL}$ to cL , and L to mL . This then implies that one movement from KL to L is 1000 thus $10 \times 10 \times 10=1 \times 10^{3}(\mathrm{i} . \mathrm{e} .1 \mathrm{KL}=$ 1000L).

Step Three / Movement Three
$1 \mathrm{KL}=1000 \mathrm{~L}$
$1 \mathrm{HL}=1000 \mathrm{dL}$
$1 \mathrm{DL}=1000 \mathrm{cL}$
$1 \mathrm{~L}=1000 \mathrm{~mL}$

Figure 8d: Four Step/ Movement


In figure 8 d , four steps were made starting from KL to the right i.e. the step from KL to $\mathrm{dL}, \mathrm{HL}$ to cL , and DL to mL . This then implies that one movement from KL to dL is 10000 thus $10 \times 10 \times 10 \times 10=1 \times 10^{4}$ (i.e. $1 \mathrm{KL}=$ 10000 dL ).

## Step Four / Movement Four

$1 \mathrm{KL}=10000 \mathrm{dL}$
$1 \mathrm{HL}=10000 \mathrm{cL}$
$1 \mathrm{DL}=10000 \mathrm{~mL}$

Figure 8e: Five Step/ Movement


In figure 8 e , five steps were made starting from KL to the right i.e. the step from KL to cL , and HL to mL . This then implies that one movement from KL to cL is 100000 thus $10 \times 10 \times 10 \times 10 \times 10=1 \times 105$ (i.e. $1 \mathrm{KL}=$ 100000 cL ).

## Step Five / Movement Four

$1 \mathrm{KL}=100000 \mathrm{cL}$
$1 \mathrm{HL}=100000 \mathrm{~mL}$

Figure 8f: Six Step/ Movement


In figure $8 f$, six steps were made starting from KL to the right i.e. the step from KL to mL . This then implies that one movement from KL to mL is 1000000 thus $10 \times 10 \times 10 \times 10 \times 10 \times 10=1 \times 106$ (i.e. $1 \mathrm{KL}=1000000 \mathrm{~mL}$ ).

```
Step Six / Movement Four
```

$1 \mathrm{KL}=1000000 \mathrm{~mL}$

Table 5
Liter-Conversion Model Summary

| Step 1 | Step 2 | Step 3 | Step 4 | Step 5 | Step 6 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $1 \mathrm{KL}=10 \mathrm{HL}$ | $1 \mathrm{KL}=100 \mathrm{DL}$ | $1 \mathrm{KL}=1000 \mathrm{~L}$ | $1 \mathrm{KL}=10000 \mathrm{dL}$ | $1 \mathrm{KL}=100000 \mathrm{cL}$ | $1 \mathrm{KL}=1000000 \mathrm{~mL}$ |
| $1 \mathrm{HL}=10 \mathrm{DL}$ | $1 \mathrm{HL}=100 \mathrm{~L}$ | $1 \mathrm{HL}=1000 \mathrm{dL}$ | $1 \mathrm{HL}=10000 \mathrm{cL}$ | $1 \mathrm{HL}=100000 \mathrm{~mL}$ |  |
| $1 \mathrm{DL}=1 \mathrm{~L}$ | $1 \mathrm{DL}=100 \mathrm{dL}$ | $1 \mathrm{DL}=1000 \mathrm{cL}$ | $1 \mathrm{DL}=10000 \mathrm{~mL}$ |  |  |
| $1 \mathrm{~L}=10 \mathrm{dL}$ | $1 \mathrm{~L}=1 \mathrm{cL}$ | $1 \mathrm{~L}=1000 \mathrm{~mL}$ |  |  |  |
| $1 \mathrm{dL}=10 \mathrm{cL}$ | $100 \mathrm{dL}=1 \mathrm{~mL}$ |  |  |  |  |
| $1 \mathrm{cL}=10 \mathrm{~mL}$ |  |  |  |  |  |

## 3. Mathematical proof of the number of step/movements on the DTML-CONVERSION model

### 3.1 Movement in one dimension

The one dimension of units in the reverse DTML-Conversion model are Length, Mass, Liter, and Time.
Figure 1: the Length (L)-Conversion Model


Figure 2: the Mass (M)-Conversion Model


Figure 3: the Liter-Conversion Model


Figure 4: the Time (T)-Conversion Model


Figure 1, 2 and 3 will be considered now and after the proving, will be applied to figure 4.
Figure 5: the proved Diagram for Reverse One Dimension-Conversion Model


Where $\mathrm{A}=\mathrm{Km}=\mathrm{Kg}=\mathrm{KL}$ (figure 1, 2 and 3 ) and $\mathrm{G}=\mathrm{mm}=\mathrm{mg}=\mathrm{mL}$ (figure 1, 2 and 3)
Let $\mathrm{y}=$ equal distance/space between alphabets (figure 5)
$\mathrm{n}=$ the number of steps / movements from the starting point, A
$\mathrm{h}=$ the index of the spaces between the alphabets i.e. $\mathrm{y}^{\mathrm{h}}$

From figure 5, n depends on y .

Mathematically,
$\mathrm{n} \propto \mathrm{y}$
Hence, $\mathrm{n}=\mathrm{ky}, \mathrm{k}=$ constant of proportionality
Let $y$ be raised to an index $h$
$n_{h}=k y^{h}, k \neq 0$, if $k=1=\left(n / y^{h}\right)=\left(n / 10^{h}\right)$
$n_{h}=y^{h}$, but $y=10$ (figure 5)
$\mathrm{n}_{\mathrm{h}}=\mathrm{y}^{\mathrm{h}}=10^{\mathrm{h}}$
$\mathrm{n}_{\mathrm{h}}=10^{\mathrm{h}}$
Application of $n=10^{h}$ to Reverse One Dimension of L, T, M and Liter- Conversion Model
> To determine the starting point to the steps/movements, then $\mathrm{h}=0$
From $\mathrm{n}_{\mathrm{h}}=10^{\mathrm{h}} \mathrm{n}_{0}=10^{0}=1$ unit, so the starting point A (figure 5 ) is 1 unit
> To determine the first movement/ step to the right of the starting point A , then $\mathrm{h}=1$
From $\mathrm{n}=10^{\mathrm{h},} \mathrm{n}_{1}=10^{1}=10$ units, so the first movement from the starting point A (figure 5) is a distance of 10 units
$>$ To determine the second movement/ step to the right of the starting point A , then $\mathrm{h}=2$
From $n_{h}=10^{\mathrm{h}} \mathrm{n}_{2}=10^{2}=10 \times 10=100$ units, so the second movement from the starting point A (figure 5) is a distance of 100 units
> To determine the third movement/ step to the right of the starting point A , then $\mathrm{h}=3$
From $n_{h}=10^{\mathrm{h}} \mathrm{n}_{3}=10^{3}=10 \times 10 \times 10=1000$ units, so the third movement from the starting point $A$ (figure 5) is a distance of 1000 units
$>$ To determine the fourth movement/ step to the right of the starting point A , then $\mathrm{h}=4$
From $n_{h}=10^{\mathrm{h},} \mathrm{n}_{4}=10^{4}=10 \times 10 \times 10 \times 10=10000$ units, so the fourth movement from the starting point A (figure 5) is a distance of 10000 units
$>$ To determine the fifth movement/ step to the right of the starting point A , then $\mathrm{h}=5$
From $\mathrm{n}_{\mathrm{h}}=10^{\mathrm{h}}, \mathrm{n}_{5}=10^{5}=10 \times 10 \times 10 \times 10 \times 10=100000$ units, so the fifth movement from the starting point A (figure 5) is a distance of 100000 units
> To determine the sixth movement/ step to the right of the starting point A , then $\mathrm{h}=6$
From $n_{h}=10^{\text {h, }} n_{6}=10^{6}=10 \times 10 \times 10 \times 10 \times 10 \times 10=1000000$ units, so the sixth movement from the starting point A (figure 5 ) is a distance of 1000000 units

Figure 4: The Reverse Time (T)-Conversion Model


Now a very closer look at figure 4 reveals that, instead of $n=10^{\mathrm{h}}$ as in figures 1,2 and 3 , $n$ now rather becomes $n_{h}=60^{\mathrm{h}}$ in figure 4.
> To determine the starting point to the steps/movements, then $\mathrm{h}=0$
From $n_{h}=60^{\mathrm{h},} \mathrm{n}_{0}=60^{\circ}=1$ unit, so the starting point Hr (figure 4) is 1 unit
> To determine the first movement/ step to the right of the starting point Hr , then $\mathrm{h}=1$
From $n_{h}=60^{\mathrm{h}} n_{1}=60^{1}=60$ units, so the first movement from the starting point Hr (figure 4) is a distance of 60 units
> To determine the second movement/ step to the right of the starting point Hr , then $\mathrm{h}=2$
From $n_{h}=60^{\mathrm{h},} n_{2}=60^{2}=60 \times 60=3600$ units, so the second movement from the starting point s (figure 4) is a distance of 3600 units

### 3.2 Movement in two dimensions

The two dimensions of units in the reverse DTML-Conversion model is Area.

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Figure 6: the reverse Area (A)-Conversion Model

| 0 | 100 | 100 | 100 | 100 | 100 | 100 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |



Figure 7: the Proved Diagram for reverse Two Dimension-Conversion Model


Let $\mathrm{y}=$ equal distance/space between alphabets (figure 7)
$\mathrm{n}=$ the number of steps / movements from the starting point $\mathrm{Km}^{2}$
$\mathrm{h}=$ the index of the spaces between the alphabets i.e. $\mathrm{y}^{\mathrm{h}}$

From figure 7, n depends on y .

Mathematically,
$\mathrm{n} \propto \mathrm{y}$
Hence, $\mathrm{n}=\mathrm{ky}, \mathrm{k}=$ constant of proportionality
Let $y$ be raised to an index $h$
$n_{h}=k y^{h}, k \neq 0$ if $k=1$
$n_{h}=y^{h}$, but $y=100$ (figure 7)
$n_{h}=y^{h}=100^{h}$
$\mathrm{n}_{\mathrm{h}}=100^{\mathrm{h}}$
Application of $n=100^{h}$ to reverse Two Dimension i.e. Area-Conversion Model
> To determine the starting point to the steps/movements, then $\mathrm{h}=0$
From $n_{h}=100^{\mathrm{h},} \mathrm{n}_{0}=100^{\circ}=1$ unit, so the starting point $\mathrm{Km}^{2}$ (figure 7) is 1 unit.
> To determine the first movement/ step to the right of the starting point $\mathrm{Km}^{2}$, then $\mathrm{h}=1$
From $n_{h}=100^{\mathrm{h}} \mathrm{n}_{1}=100^{1}=100$ units, so the first movement from the starting point $\mathrm{Km}^{2}$ (figure 7) is a distance of 100 units.
> To determine the second movement/ step to the right of the starting point $\mathrm{Km}^{2}$, then $\mathrm{h}=2$
From $n_{h}=100^{\mathrm{h}} \mathrm{n}_{2}=100^{2}=100 \times 100=10000$ units, so the second movement from the starting point $\mathrm{Km}^{2}$ (figure 7) is a distance of 10000 units.
> To determine the third movement/ step to the right of the starting point $\mathrm{Km}^{2}$, then $\mathrm{h}=3$
From $n_{h}=100^{\mathrm{h}} n_{3}=100^{3}=100 \times 100 \times 100=1000000$ units, so the third movement from the starting point $\mathrm{Km}^{2}$ (figure 7) is a distance of 1000000 units.
$>$ To determine the fourth movement/ step to the right of the starting point $\mathrm{Km}^{2}$, then $\mathrm{h}=4$
From $n_{h}=100^{\mathrm{h},} \mathrm{n}_{4}=100^{4}=100 \times 100 \times 100 \times 100=100000000$ units, so the fourth movement from the starting point $\mathrm{Km}^{2}$ (figure 7) is a distance of 100000000 units.

The reverse DTML-conversion model with its mathematical formulas
$>$ To determine the fifth movement/ step to the right of the starting point $\mathrm{Km}^{2}$, then $\mathrm{h}=5$
From $n_{h}=100^{\mathrm{h}} \mathrm{n}_{5}=100^{5}=100 \times 100 \times 100 \times 100 \times 100=10000000000$ units, so the fifth movement from the starting point $\mathrm{Km}^{2}$ (figure 7) is a distance of 10000000000 units.
> To determine the sixth movement/ step to the right of the starting point $\mathrm{Km}^{2}$, then $\mathrm{h}=6$
From $n_{h}=100^{\text {h, }} n_{6}=100^{6}=100 \times 100 \times 100 \times 100 \times 100 \times 100=1000000000000$ units, so the sixth movement from the starting point $\mathrm{Km}^{2}$ (figure 7) is a distance of 1000000000000 units.

### 3.3 Movement in three dimensions

The three dimensions of units in the DTML-Conversion model is Volume.

## Figure 8: the Volume (V)-Conversion Model

| 0 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |



Figure 9: the Proved Diagram for reverse Three Dimension-Conversion Model


Let $y=$ equal distance/space between alphabets (figure 9)
$\mathrm{n}=$ the number of steps / movements from the starting point, $\mathrm{Km}^{3}$
$h=$ the index of the spaces between the alphabets i.e. $\mathrm{y}^{\mathrm{h}}$

From figure 9, n depends on y .
Mathematically,
$\mathrm{n} \propto \mathrm{y}$
Hence, $\mathrm{n}=\mathrm{ky}, \mathrm{k}=$ constant of proportionality
Let $y$ be raised to an index $h$
$n_{h}=k y^{h}, k \neq 0$, if $k=1$
$n_{h}=y^{\text {h }}$, but $y=1000$ (figure 9)
$n_{h}=y^{h}=1000^{h}$
$\mathrm{n}_{\mathrm{h}}=1000^{\mathrm{h}}$

### 3.4 Application of $n=1000^{h}$ to Three Dimension i.e. Volume-Conversion Model

> To determine the starting point to the steps/movements, then $\mathrm{h}=0$
From $\mathrm{nh}=1000 \mathrm{~h}, \mathrm{n} 0=10000=1$ unit, so the starting point Km 3 (figure 9) is 1 unit.
$>$ To determine the first movement/ step to the right of the starting point $\mathrm{Km}^{3}$, then $\mathrm{h}=1$
From $n_{h}=1000^{\mathrm{h}} \mathrm{n}_{1}=1000^{1}=1000$ units, so the first movement from the starting point $\mathrm{Km}^{3}$ (figure 9) is a distance of 1000 units.
$>$ To determine the second movement/ step to the right of the starting point $\mathrm{Km}^{3}$, then $\mathrm{h}=2$
From $\mathrm{nh}=1000 \mathrm{~h}, \mathrm{n} 2=1000^{2}=1000 \times 1000=1000000$ units, so the second movement from the starting point Km3 (figure 9) is a distance of 1000000 units.
> To determine the third movement/ step to the right of the starting point $\mathrm{Km}^{3}$, then $\mathrm{h}=3$
From $\mathrm{nh}=1000 \mathrm{~h}, \mathrm{n} 3=1000^{3}=1000 \times 1000 \times 1000=1000000000$ units, so the third movement from the starting point $\mathrm{Km}^{3}$ (figure 9) is a distance of 1000000000 units.
$>$ To determine the fourth movement/ step to the right of the starting point $\mathrm{Km}^{3}$, then $\mathrm{h}=4$
From $\mathrm{nh}=1000 \mathrm{~h}, \mathrm{n} 4=1000^{4}=1000 \times 1000 \times 1000 \times 1000=1000000000000$ units, so the fourth movement from the starting point Km 3 (figure 9) is a distance of 1000000000000 units.
$>$ To determine the fifth movement/ step to the right of the starting point $\mathrm{Km}^{3}$, then $\mathrm{h}=5$
From nh $=1000 \mathrm{~h}, \mathrm{n} 5=1000^{5}=1000 \times 1000 \times 1000 \times 1000 \times 1000=1000000000000000$ units, so the fifth movement from the starting point Km3 (figure 9) is a distance of 1000000000000000 units.
> To determine the sixth movement/ step to the right of the starting point $\mathrm{Km}^{3}$, then $\mathrm{h}=6$
From $\mathrm{nh}=1000 \mathrm{~h}, \mathrm{n} 6=100^{6}=1000 \times 1000 \times 1000 \times 1000 \times 1000 \times 1000=1000000000000000000$ units, so the sixth movement from the starting point Km3 (figure) is a distance of 1000000000000000000 units.

## 4. Mathematical proof of converting within units with the reverse

## DTML-CONVERSION MODEL

Let $\mathrm{y}=$ the given unit
$\mathrm{c}=$ the unit in which the given unit is to be converted
$\mathrm{n}=$ the number of steps / movements from the starting point
$d=$ the dimension of a quantity

If, $y$ depends on $n$.
Mathematically,
$y \propto n$
Hence, $y=c n, c=$ constant of proportionality
Let $y$ be raised to an index $d$
$y=\mathrm{cn}^{\mathrm{d}} ; \mathrm{c} \neq 0$, if for one dimension $\mathrm{d}=1$, for two dimension $\mathrm{d}=2$, and for three dimension $\mathrm{d}=3$
$\mathrm{y}=\mathrm{cn}^{\mathrm{d}}$, if $\mathrm{d}=1$ (one dimension)
$\mathrm{y}=\mathrm{cn}^{\mathrm{d}}=\mathrm{cn}^{1}=\mathrm{cn}$ (one dimension)
$\mathrm{y}=\mathrm{cn}^{\mathrm{d}}$, if $\mathrm{d}=2$, then $\mathrm{y}=\mathrm{cn}^{2}$ (two dimension; n is the number of steps/movement in the one dimension-conversion model)
$\mathrm{y}=\mathrm{cn}^{\mathrm{d}}=\mathrm{cn}^{2}=\mathrm{cn}$ (two dimension; n is the actual number of steps in the two dimension-conversion model)
$\mathrm{y}=\mathrm{cn}^{\mathrm{d}}$, if $\mathrm{d}=3$, then $\mathrm{y}=\mathrm{cn}^{3}$ (three dimension; n is the number of steps/movement in the one dimension-conversion model)
$\mathrm{y}=\mathrm{cn}^{\mathrm{d}}=\mathrm{cn}^{3}=\mathrm{cn}$ (three dimension; n is the actual number of steps in the three dimension-conversion model)

### 4.1 Application of $y=c n^{d}$ to One Dimension of L, M, T and Liter- Conversion Model

$y=c n d$, for one dimension, $d=1$
$y=c n$

## Example 1

Convert 2 cm to m .
Data: $\mathrm{y}=$ the given unit (i.e. 2 cm ), $\mathrm{n}=$ number of steps (two steps, $\mathrm{n} 2=100$ i.e. converting from cm to m is a two step/movement i.e. $\mathrm{n} 2=100$ ), $\mathrm{c}=$ the unit in which the given unit is to be converted.

## Solution

$\mathrm{y}=\mathrm{cn}$
$2 \mathrm{~cm}=\mathrm{c} \times 100 \mathrm{~cm} / \mathrm{m}$
$2 \mathrm{~cm} / 100 \mathrm{~cm} \times 1 \mathrm{~m}=\mathrm{c}$
$0.02 \mathrm{~m}=\mathrm{c}$

## Example 2

Change 0.001 g to dg

## Data:

$\mathrm{y}=0.001 \mathrm{~g}, \mathrm{n}=1 \div 10$ (moving from g to dg is one movement but it is the reverse of dg to g , hence instead of $\mathrm{n}=10$, the reverse is $\mathrm{n}=(1 \div 10), \mathrm{c}=$ ?

## Solution

```
y = cn
```

$0.001 \mathrm{~g}=\mathrm{c} \times 1 / 10 \mathrm{~g} / \mathrm{dg}$
$0.001 \mathrm{~g} /(1 / 10 \mathrm{~g}) \times 1 \mathrm{dg}=\mathrm{c}$
$0.001 \times 10 \mathrm{dg}=\mathrm{c}$
$0.01 \mathrm{dg}=\mathrm{c}$

## Example 3

Convert 534 kL to mL

## Data:

$\mathrm{y}=534 \mathrm{~kL}, \mathrm{n}=1 \div 1000000$ (moving from kL to mL is six steps/movement but it is the reverse of mL to kL , hence instead of $n=1000000$, the reverse is $n=(1 \div 1000000), \mathrm{c}=$ ?

## Solution

```
y = cn
534kL = c }\times1/1000000kL/m
534kL/ (1/1000000kL/mL) = c
534\times1000000mL = c
534000000mL = c
```


## Example 4

Change 200s to hr.

## Data:

$\mathrm{y}=200 \mathrm{~s}, \mathrm{n}=3600$ (moving from s to hr is two steps/movements), $\mathrm{c}=$ ?

## Solution

$$
\begin{aligned}
& y=\mathrm{cn} \\
& 200 \mathrm{~s}=\mathrm{c} \times(3600) \mathrm{s} / \mathrm{hr} \\
& 200 \mathrm{~s} / 3600 \mathrm{~s} \times 1 \mathrm{hr}=\mathrm{c} \\
& (200 / 3600) \times 1 \mathrm{hr}=\mathrm{c} \\
& 0.056 \mathrm{hr}=\mathrm{c}
\end{aligned}
$$

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4.2 Application of $y=\mathrm{cn}^{2}$ to Two Dimension i.e. Area (A) - Conversion Model

## Example 1

Convert $2 \mathrm{~cm}^{2}$ to $\mathrm{m}^{2}$

## Data:

$y=$ the given unit (i.e. $2 \mathrm{~cm}^{2}$ ), $n=$ number of steps (two steps, $n_{2}=10000=n_{2 \#}^{2}=100^{2}$ i.e. $n_{2 \#}$ is the two step / movement in the one dimension-conversion model, also converting from $\mathrm{cm}^{2}$ to $\mathrm{m}^{2}$ is a two step/movement i.e. $\mathrm{n}_{2}=10000$ ), $\mathrm{c}=$ the unit in which the given unit is to be converted.

## Solution

By using $\mathrm{n}_{2 \#}=100$ (two step / movement in the one dimension-conversion model)
$y=\mathrm{cn}^{\mathrm{d}}$, for two dimension, $\mathrm{d}=2$
$\mathrm{y}=\mathrm{cn}^{\mathrm{d}}=\mathrm{cn}^{2}$
$2 \mathrm{~cm}^{2}=\mathrm{c} \times \mathrm{n}_{2 \#}^{2}=\mathrm{c} \times 100^{2} \mathrm{~cm}^{2} / \mathrm{m}^{2}$
$2 \mathrm{~cm}^{2}=\mathrm{c} \times 10000 \mathrm{~cm}^{2} / \mathrm{m}^{2}$
$\left(2 / 10000 \mathrm{~cm}^{2} / \mathrm{cm}^{2}\right) \times 1 \mathrm{~m}^{2}=\mathrm{c}$
$2 / 10000 \mathrm{~m}^{2}=\mathrm{c}$
$0.0002 \mathrm{~m}^{2}=\mathrm{c}$
By using $\mathrm{n}_{2}=10000$ (actual two step / movement in the two dimension-conversion model)
$\mathrm{y}=\mathrm{cn}_{2}$
$2 \mathrm{~cm}^{2}=\mathrm{c} \times 10000 \mathrm{~cm}^{2} / \mathrm{m}^{2}$
$\left(2 / 10000 \mathrm{~cm}^{2} / \mathrm{cm}^{2}\right) \times 1 \mathrm{~m}^{2}=\mathrm{c}$
$0.0002 \mathrm{~m}^{2}=\mathrm{c}$

## Example 2

Convert $2 \mathrm{~m}^{2}$ to $\mathrm{cm}^{2}$

## Data:

$\mathrm{y}=$ the given unit (i.e. $2 \mathrm{~m}^{2}$ ), $\mathrm{n}=$ number of steps (two steps in a reverse direction i.e. from $\mathrm{m}^{2}$ to $\mathrm{cm}^{2}, \mathrm{n}_{2}=$ $1 \div 10000=n_{2 \#}^{2}=(1 \div 100)^{2}$ i.e. $n_{2 \#}$ is the two step / movement in the one dimension-conversion model, also converting from $\mathrm{m}^{2}$ to $\mathrm{cm}^{2}$ is a two step/movement but in a reversed direction i.e. $\mathrm{n}_{2}=1 \div 10000, \mathrm{c}=$ the unit in which the given unit is to be converted.

## Solution

By using $n_{2 \#}=2 / 100$ (two step / movement in the one dimension-conversion model)
$y=\mathrm{cn}^{\mathrm{d}}$, for two dimension, $\mathrm{d}=2$
$\mathrm{y}=\mathrm{cn}^{\mathrm{d}}=\mathrm{cn}^{2}$
$2 \mathrm{~m}^{2}=\mathrm{c} \times \mathrm{n}_{2 \#}{ }^{2}=\mathrm{c} \times\left(1 \div 100^{2}\right) \mathrm{m}^{2} / \mathrm{cm}^{2}$
$2 \mathrm{~m}^{2}=\mathrm{c} \times 10000 \mathrm{~m}^{2} / \mathrm{cm}^{2}$
$\left[2 /(1 / 100)^{2} \mathrm{~m}^{2} / \mathrm{m}^{2}\right] \times 1 \mathrm{~cm}^{2}=\mathrm{c}$
$2 \times 10000 \mathrm{~cm}^{2}=\mathrm{c}$
$20000 \mathrm{~cm}^{2}=\mathrm{c}$

The reverse DTML-conversion model with its mathematical formulas
By using $\mathrm{n}_{2}=1 / 1000$ (actual two step / movement in the two dimension-conversion model)
$\mathrm{y}=\mathrm{cn}_{2}$
$2 \mathrm{~m}^{2}=\mathrm{c} \times(1 / 1000) \mathrm{m}^{2} / \mathrm{cm}^{2}$
$\left(2 \mathrm{~m}^{2} \times 10000 \mathrm{~m}^{2}\right) \times 1 \mathrm{~cm}^{2}=\mathrm{c}$
$20000 \mathrm{~cm}^{2}=\mathrm{c}$

### 4.3 Application of $y=$ cn3 to Three Dimension i.e. Volume (V)- Conversion Model

## Example 1

Convert $2 \mathrm{~cm}^{3}$ to $\mathrm{m}^{3}$

## Data:

$y=$ the given unit (i.e. $2 \mathrm{~cm}^{3}$ ), $n=$ number of steps (two steps, $n_{2}=1000000$ i.e. $1000 \times 1000=n_{2 \#}^{3}=100^{3}$ i.e. $n_{2 \#}$ is the two step / movement in the one dimension-conversion model, also converting from $\mathrm{cm}^{3}$ to $\mathrm{m}^{3}$ is a two step/movement i.e. $\mathrm{n}_{2}=1000000, \mathrm{c}=$ the unit in which the given unit is to be converted.

## Solution

By using $\mathrm{n}_{2 \#}=100$ (two step / movement in the one dimension-conversion model)
$y=\mathrm{cn}^{\mathrm{d}}$, for three dimension, $\mathrm{d}=3$
$\mathrm{y}=\mathrm{cn}^{\mathrm{d}}=\mathrm{cn}^{3}$
$2 \mathrm{~cm}^{3}=\mathrm{c} \times \mathrm{n}_{2 \#}{ }^{3}=\mathrm{c} \times 100^{3} \mathrm{~cm}^{3} / \mathrm{m}^{3}$
$2 \mathrm{~cm}^{3}=\mathrm{c} \times 1000000 \mathrm{~cm}^{3} / \mathrm{m}^{3}$
$\left(2 / 1000000 \mathrm{~cm}^{3} / \mathrm{cm}^{3}\right) \times 1 \mathrm{~m}^{3}=\mathrm{c}$
$2 / 1000000 \mathrm{~m}^{3}=\mathrm{c}$
$0.000002 \mathrm{~m}^{3}=\mathrm{c}$
By using $\mathrm{n}_{2}=1000000$ (actual two step / movement in the three dimension-conversion model)
$\mathrm{y}=\mathrm{cn}_{2}$
$2 \mathrm{~cm}^{3}=\mathrm{c} \times 1000000 \mathrm{~cm}^{3} / \mathrm{m}^{3}$
$\left(2 / 1000000 \mathrm{~cm}^{3} / \mathrm{cm}^{3}\right) \times 1 \mathrm{~m}^{3}=\mathrm{c}$
$0.000002 \mathrm{~m}^{3}=\mathrm{c}$

## Example 2

Convert $2 \mathrm{~m}^{3}$ to $\mathrm{cm}^{3}$

## Data:

$\mathrm{y}=$ the given unit (i.e. $2 \mathrm{~m}^{3}$ ), $\mathrm{n}=$ number of steps (two steps but in a reversed movement, $\mathrm{n}_{2}=1 \div 1000000$ i.e. $=$ $\mathrm{n}_{2 \#}{ }^{3}=1 \div 100^{3}$ i.e. $\mathrm{n}_{2 \#}$ is the reversed two step / movement in the one dimension-conversion model, also converting from $\mathrm{m}^{3}$ to $\mathrm{cm}^{3}$ is a reversed two step/movement i.e. $\mathrm{n}_{2}=1000000, \mathrm{c}=$ the unit in which the given unit is to be converted.

## Solution

By using $n_{2 \#}=1 / 100$ (two step / movement in the one dimension-conversion model)
$y=\mathrm{cn}^{\mathrm{d}}$, for three dimension, $\mathrm{d}=3$

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```
\(y=\mathrm{cn}^{\mathrm{d}}=\mathrm{cn}^{3}\)
\(2 \mathrm{~m}^{3}=\mathrm{c} \times \mathrm{n}_{2 \#}{ }^{3}=\mathrm{c} \times\left(1 \div 100^{3}\right) \mathrm{m}^{3} / \mathrm{cm}^{3}\)
\(2 \mathrm{~m}^{3}=\mathrm{c} \times 1 / 1000000 \mathrm{~cm}^{3} / \mathrm{m}^{3}\)
\(\left(2 \mathrm{~m}^{3} \times 1000000 \mathrm{~m}^{3}\right) \times 1 \mathrm{~cm}^{3}=\mathrm{c}\)
\(2 \times 1000000 \mathrm{~cm}^{3}=\mathrm{c}\)
\(2000000 \mathrm{~cm}^{3}=\mathrm{c}\)
By using \(\mathrm{n}_{2}=1 / 1000000\) (actual two step / movement in the three dimension-conversion model)
\(\mathrm{y}=\mathrm{cn}_{2}\)
\(2 \mathrm{~m}^{3}=\mathrm{c} \times 1 / 1000000 \mathrm{~m}^{3} / \mathrm{cm}^{3}\)
\(\left(2 \mathrm{~m}^{3} \times 1000000 \mathrm{~m}^{3}\right) \times 1 \mathrm{~cm}^{3}=\mathrm{c}\)
\(2000000 \mathrm{~cm}^{3}=\mathrm{c}\)
```


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